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SPG-Separation Axioms

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Abstract

In this paper by using spg-open sets we define almost spg-normality and mild spg-normality also we continue the study of further properties of spg-normality. We show that these three axioms are regular open hereditary. We also define the class of almost spg-irresolute mappings and show that spg-normality is invariant under almost spg-irresolute M-spg-open continuous surjection.

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Introduction

In 1967, A. Wilansky has introduced the concept of US spaces. In 1968, C.E. Aull studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, in 1982, S.P. Arya et al have introduced and studied the concept of semi-US spaces and also they made study of s-convergence, sequentially semi-closed sets, sequentially s-compact notions. G.B. Navlagi studied P-Normal Almost-P-Normal, Mildly-P-Normal and Pre-US spaces. Recently S. Balasubramanian and P.Aruna Swathi Vyjayanthi studied *v*-Normal Almost- *v*-Normal, Mildly-*v*-Normal and *v*-US spaces. Inspired with these we introduce spg-Normal Almost- spg-Normal, Mildly- spg-Normal, spg-US, spg-S_1 and spg-S_2. Also we examine spg-convergence, sequentially spg-compact, sequentially spg-continuous maps, and sequentially sub spg-continuous maps in the context of these new concepts. All notions and symbols which are not defined in this paper may be found in the appropriate references. Throughout the paper X and Y denote Topological spaces on which no separation axioms are assumed explicitly stated.

Preliminaries

Definition 2.1: $A \subset X$ is called

(i) g-closed if cl $A \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) pg-closed if $pcl(A) \subseteq U$ whenever $A \subseteq U$ and U is preopen in X.
(iii) spg-closed if $spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is sp-open in X.

Definition 2.2: A space X is said to be
(i) T_1 (T_2) if for any $x \neq y$ in X, there exist (disjoint) open sets U; V in X such that $x \in U$ and $y \in V$.
(ii) weakly Hausdorff if each point of X is the intersection of regular closed sets of X.
(iii) normal[resp: mildly normal] if for any pair of disjoint [resp: regular-closed]closed sets F_1 and F_2 , there exist disjoint open sets U and V such that $F_1 \subset U$ and $F_2 \subset V$.

(iv) almost normal if for each closed set A and each regular closed set B such that $A \cap B = \phi$, there exist disjoint open sets U and V such that $A \subset U$ and $B \subset V$.
(v) weakly regular if for each pair consisting of a regular closed set A and a point x such that $A \cap \{x\} = \phi$, there exist disjoint open sets U and V such that $x \in U$ and $A \subset V$.
(vi) A subset A of a space X is S-closed relative to X if every cover of A by semiopen sets of X has a finite subfamily whose closures cover A.

(vii) R_0 if for any point x and a closed set F with $x \notin F$ in X, there exists a open set G containing F but not x.
(viii) R_1 iff for x, $y \in X$ with $cl\{x\} \neq cl\{y\}$, there exist disjoint open sets U and V such that $cl\{x\} \subset U$, $cl\{y\} \subset V$.
(ix) US-space if every convergent sequence has exactly one limit point to which it converges.
(x) pre-US space if every pre-convergent sequence has exactly one limit point to which it converges.
(xi) pre-S_1 if it is pre-US and every sequence $<x_n>$ pre-converges with subsequence of $<x_n>$ pre-side points.

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(xii) pre-S_2 if it is pre-US and every sequence $<x_n>$ in X pre-converges which has no pre-side point.
(xiii) is weakly countable compact if every infinite subset of X has a limit point in X.
(xiv) Baire space if for any countable collection of closed sets with empty interior in X, their union also has empty interior in X.

Definition 2.3: Let $A\subset X$. Then a point x is said to be a
(i) limit point of *A* if each open set containing x contains some point y of *A* such that $x \neq y$.
(ii) T_0–limit point of *A* if each open set containing x contains some point y of *A* such that cl{x} ≠ cl{y}, or equivalently, such that they are topologically distinct.
(iii) *pre*-T_0–limit point of *A* if each open set containing x contains some point y of *A* such that *p*cl{x} ≠ *p*cl{y}, or equivalently, such that they are topologically distinct.

Note 1: Recall that two points are topologically distinguishable or distinct if there exists an open set containing one of the points but not the other; equivalently if they have disjoint closures. In fact, the T_0–axiom is precisely to ensure that any two distinct points are topologically distinct.

Example 1: Let X = {a, b, c, d} and τ = {{a}, {b, c}, {a, b, c}, X, ϕ}. Then b and c are the limit points but not the T_0–limit points of the set {b, c}. Further d is a T_0–limit point of {b, c}.

Example 2: Let X = (0, 1) and τ = {ϕ, X, and U_n = (0, 1–1⁄n), n = 2, 3, 4,. . . }. Then every point of X is a limit point of X. Every point of X~U_2 is a T_0–limit point of X, but no point of U_2 is a T_0–limit point of X.

Definition 2.4: A set *A* together with all its T_0–limit points will be denoted by T_0–cl*A*.

Note 2: i. Every T_0–limit point of a set *A* is a limit point of the set but the converse is not true in general.
ii. In T_0–space both are same.

Note 3: R_0–axiom is weaker than T_1–axiom. It is independent of the T_0–axiom. However $T_1 = R_0+T_0$

Note 4: Every countable compact space is weakly countable compact but converse is not true in general. However, a T_1–space is weakly countable compact iff it is countable compact.

spg-T_0 LIMIT POINT

Definition 3.01: In X, a point x is said to be a spg-T_0–limit point of *A* if each spg-open set containing x contains some point y of *A* such that *spgcl*{x} ≠ *spgcl*{y}, or equivalently; such that they are topologically distinct with respect to spg-open sets.

Note 5: *regular open set* ⇒ *open set* ⇒ pre-*open set* ⇒ spg-*open set we have*
r-T_0–limit point ⇒ *T_0–limit point* ⇒ pre-*T_0–limit point* ⇒ spg-*T_0–limit point*

Example 3: Let X = {a, b, c, d} and τ = {ϕ, {b}, {a, b}, {b, c}, {a, b, c}, X}. For A = {a, b, c}, a and b are *spg-T_0*–limit point.

Definition 3.02: A set A together with all its spg-T_0–limit points is denoted by T_0-*spgcl*(A)

Lemma 3.01: *If x is a* spg-*T_0–limit point of a set A then x is* spg-*limit point of A.*

Lemma 3.02:
(i) If X is spg-*T_0–space then every* spg-*T_0–limit point and every* spg-*limit point are equivalent.*
(ii)If X is r-T_0–space then every spg-*T_0–limit point and every* spg-*limit point are equivalent.*

Theorem 3.03: *For $x \neq y \in X$,*
(i) x is a spg-*T_0–limit point of {y} iff x∉spgcl{y} and y∈spgcl{x}.*
(ii) x is not a spg-*T_0–limit point of {y} iff either x∈spgcl{y}or spgcl{x} = spgcl{y}.*
(iii) x is not a spg-*T_0–limit point of {y} iff either x∈spgcl{y}or y∈spgcl{x}.*

Corollary 3.04:
(i) If x is a spg-*T_0–limit point of {y}, then y cannot be a* spg-*limit point of {x}.*
(ii) If spgcl{x} = spgcl{y}, then neither x is a spg-*T_0–limit point of {y} nor y is a* spg-*T_0–limit point of {x}.*
(iii) If a singleton set A has no spg-*T_0–limit point in X, then spgclA = spgcl{x} for all x∈ spgcl{A}.*

Lemma 3.05: *In X, if x is a* spg-*limit point of a set A, then in each of the following cases x becomes* spg-*T_0–limit point of A ({x} ≠ A).*
(i) spgcl{x} ≠ spgcl{y} for y∈A, x ≠ y.
(ii) spgcl{x} = {x}
(iii) X is a spg-*T_0–space.*
(iv) A~{x} is spg-*open*

spg-T_0 AND *spg*-R_i AXIOMS, i = 0,1

In view of Lemma 3.6(iii), *spg*-T_0–axiom implies the equivalence of the concept of limit point of a set with that of *spg*-T_0–limit point of the set. But for the converse, if x∈ *spg*cl{y} then *spg*cl{x} ≠ *spg*cl{y} in general, but if x is a *spg*-T_0–limit point of {y}, then *spg*cl{x} = *spg*cl{y}

Lemma 4.01: *In a space X, a limit point x of {y} is a spg-T_0–limit point of {y} iff spgcl{x} ≠ spgcl{y}.*

This lemma leads to characterize the equivalence of *spg*-T_0–limit point and *spg*-limit point of a set as the *spg*-T_0–axiom.

Theorem 4.02: *The following conditions are equivalent:*
- *(i) X is a spg-T_0 space*
- *(ii) Every spg-limit point of a set A is a spg-T_0–limit point of A*
- *(iii) Every r-limit point of a singleton set {x} is a spg-T_0–limit point of {x}*
- *(iv) For any x, y in X, x ≠ y if x∈ spgcl{y}, then x is a spg-T_0–limit point of {y}*

Note 6: In a *spg*-T_0–space X if every point of X is a r-limit point of X, then every point of X is *spg*-T_0–limit point of X. But a space X in which each point is a *spg*-T_0–limit point of X is not necessarily a *spg*-T_0–space

Theorem 4.03: *The following conditions are equivalent:*
- *(i) X is a spg-R_0 space*
- *(ii) For any x, y in X, if x∈ spgcl{y}, then x is not a spg-T_0–limit point of {y}*
- *(iii) A point spg-closure set has no spg-T_0–limit point in X*
- *(iv) A singleton set has no spg-T_0–limit point in X.*

Theorem 4.04: *In a spg-R_0 space X, a point x is spg-T_0–limit point of A iff every spg-open set containing x contains infinitely many points of A with each of which x is topologically distinct*

Theorem 4.05: *X is spg-R_0 space iff a set A of the form A = ∪ spgcl{$x_{i\ i=1\ to\ n}$} a finite union of point closure sets has no spg-T_0–limit point.*

If *spg*-R_0 space is replaced by rR_0 space in the above theorem, we have the following corollaries:

Corollary 4.06: *The following conditions are equivalent:*
- *(i) X is a r-R_0 space*
- *(ii) For any x, y in X, if x∈ spgcl{y}, then x is not a spg-T_0–limit point of {y}*
- *(iii) A point spg-closure set has no spg-T_0–limit point in X*
- *(iv) A singleton set has no spg-T_0–limit point in X.*

Corollary 4.07: *In an rR_0–space X,*
- *(i) If a point x is rT_0–limit point of a set then every spg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.*
- *(ii) If a point x is spg-T_0–limit point of a set then every spg-open set containing x contains infinitely many points of A with each of which x is topologically distinct.*
- *(iii)If A = ∪ spgcl{$x_{i,\ i=1\ to\ n}$} a finite union of point closure sets has no spg-T_0–limit point.*
- *(iv)If X = ∪ spgcl{$x_{i,\ i=1\ to\ n}$} then X has no spg-T_0–limit point.*

Various characteristic properties of *spg*-T_0–limit points studied so far is enlisted in the following theorem.

Theorem 4.08: *In a spg-R_0–space, we have the following:*
- *(i) A singleton set has no spg-T_0–limit point in X.*
- *(ii) A finite set has no spg-T_0–limit point in X.*
- *(iii) A point spg-closure has no set spg-T_0–limit point in X*
- *(iv) A finite union point spg-closure sets have no set spg-T_0–limit point in X.*
- *(v) For x, y∈ X, x∈T_0– spgcl{y} iff x = y.*
- *(vi) For any x, y∈ X, x ≠ y iff neither x is spg-T_0–limit point of {y}nor y is spg-T_0–limit point of {x}*
- *(vii) For any x, y∈ X, x ≠ y iff T_0– spgcl{x} ∩T_0– spgcl{y} = ϕ.*
- *(viii)Any point x∈X is a spg-T_0–limit point of a set A in X iff every spg-open set containing x contains infinitely many points of A with each which x is topologically distinct.*

Theorem 4.09: *X is spg-R_1 iff for any spg-open set U in X and points x, y such that x∈X~U, y∈U, there exists a spg-open set V in X such that y∈V⊂U, x∉V.*

Lemma 4.10: *In spg-R_1 space X, if x is a spg-T_0–limit point of X, then for any non empty spg-open set U, there exists a non empty spg-open set V such that V⊂U, x∉ spgcl(V).*

Lemma 4.11: *In a spg- regular space X, if x is a spg-T_0–limit point of X, then for any non empty spg-open set U, there exists a non empty spg-open set V such that spgcl(V)⊂U, x∉ spgcl(V).*

Corollary 4.12: *In a regular space X,*

(i) If x is a spg-T_0–limit point of X, then for any non empty spg-open set U, there exists a non empty spg-open set V such that spgcl(V)⊂U, x∉ spgcl(V).

(ii) If x is a T_0–limit point of X, then for any non empty spg-open set U, there exists a non empty spg-open set V such that spgcl(V)⊂U, x∉ spgcl(V).

Theorem 4.13: *If X is a spg-compact spg-R_1-space, then X is a Baire Space.*

Proof: Let $\{A_n\}$ be a countable collection of *spg*-closed sets of X, each A_n having empty interior in X. Take A_1, since A_1 has empty interior, A_1 does not contain any *spg*-open set say U_0. Therefore we can choose a point $y\in U_0$ such that $y\notin A_1$. For X is *spg*-regular, and $y\in(X\sim A_1)\cap U_0$, a *spg*-open set, we can find a *spg*-open set U_1 in X such that $y\in U_1$, $spgcl(U_1)\subset(X\sim A_1)\cap U_0$. Hence U_1 is a non empty *spg*-open set in X such that $spgcl(U_1)\subset U_0$ and $spgcl(U_1)\cap A_1 = \phi$. Continuing this process, in general, for given non empty *spg*-open set U_{n-1}, we can choose a point of U_{n-1} which is not in the *spg*-closed set A_n and a *spg*-open set U_n containing this point such that $spgcl(U_n)\subset U_{n-1}$ and $spgcl(U_n)\cap A_n = \phi$. Thus we get a sequence of nested non empty *spg*-closed sets which satisfies the finite intersection property. Therefore $\cap\ spgcl(U_n) \neq \phi$. Then some $x\in\cap\ spgcl(U_n)$ which in turn implies that $x\in U_{n-1}$ as $spgcl(U_n)\subset U_{n-1}$ and $x\notin A_n$ for each n.

Corollary 4.14: *If X is a compact spg-R_1-space, then X is a Baire Space.*

Corollary 4.15: *Let X be a spg-compact spg-R_1-space. If $\{A_n\}$ is a countable collection of spg-closed sets in X, each A_n having non-empty spg-interior in X, then there is a point of X which is not in any of the A_n.*

Corollary 4.16: *Let X be a spg-compact R_1-space. If $\{A_n\}$ is a countable collection of spg-closed sets in X, each A_n having non-empty spg- interior in X, then there is a point of X which is not in any of the A_n.*

Theorem 4.17: *Let X be a non empty compact spg-R_1-space. If every point of X is a spg-T_0–limit point of X then X is uncountable.*

Proof: Since X is non empty and every point is a *spg*-T_0-limit point of X, X must be infinite. If X is countable, we construct a sequence of *spg*- open sets $\{V_n\}$ in X as follows:

Let $X = V_1$, then for x_1 is a *spg*-T_0-limit point of X, we can choose a non empty *spg*-open set V_2 in X such that $V_2\subset V_1$ and $x_1\notin spgclV_2$. Next for x_2 and non empty *spg*-open set V_2, we can choose a non empty *spg*-open set V_3 in X such that $V_3\subset V_2$ and $x_2\notin spgclV_3$. Continuing this process for each x_n and a non empty *spg*-open set V_n, we can choose a non empty *spg*-open set V_{n+1} in X such that $V_{n+1}\subset V_n$ and $x_n\notin spgclV_{n+1}$.

Now consider the nested sequence of *spg*-closed sets $spgclV_1\supset spgclV_2\supset spgclV_3\supset\ldots\ldots\supset spgclV_n\supset\ldots$ Since X is *spg*-compact and $\{spgclV_n\}$ the sequence of *spg*-closed sets satisfies finite intersection property. By Cantors intersection theorem, there exists an x in X such that $x\in spgclV_n$. Further $x\in X$ and $x\in V_1$, which is not equal to any of the points of X. Hence X is uncountable.

Corollary 4.18: *Let X be a non empty spg-compact spg-R_1-space. If every point of X is a spg-T_0–limit point of X then X is uncountable*

spg–T_0-IDENTIFICATION SPACES AND *spg*–SEPARATION AXIOMS

Definition 5.01: Let (X, τ) be a topological space and let $\Re$ be the equivalence relation on X defined by $x\Re y$ iff $spgcl\{x\} = spgcl\{y\}$

Problem 5.02: show that $x\Re y$ iff $spgcl\{x\} = spgcl\{y\}$ is an equivalence relation

Definition 5.03: The space $(X_0, Q(X_0))$ is called the spg-T_0–identification space of (X, τ), where X_0 is the set of equivalence classes of $\Re$ and $Q(X_0)$ is the decomposition topology on X_0.
Let P_X: $(X,\tau)\rightarrow(X_0, Q(X_0))$ denote the natural map

Lemma 5.04: *If $x\in X$ and $A\subset X$, then $x\in spgclA$ iff every* spg-*open set containing x intersects A.*

Theorem 5.05: *The natural map $P_X:(X,\tau)\rightarrow(X_0, Q(X_0))$ is closed, open and $P_X^{-1}(P_X(O)) = O$ for all $O\in PO(X,\tau)$ and $(X_0, Q(X_0))$ is* spg-*T_0*

Proof: Let $O\in PO(X,\tau)$ and let $C\in P_X(O)$. Then there exists $x\in O$ such that $P_X(x) = C$. If $y\in C$, then $spgcl\{y\} = spgcl\{x\}$, which, by lemma, implies $y\in O$. Since $\tau\subset PO(X,\tau)$, then $P_X^{-1}(P_X(U)) = U$ for all $U\in\tau$, which implies P_X is closed and open.

Let $G, H\in X_0$ such that $G\neq H$; let $x\in G$ and $y\in H$. Then $spgcl\{x\}\neq spgcl\{y\}$, which implies $x\notin spgcl\{y\}$ or

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y∉*spgcl*{x}, say x∉*spgcl*{y}. Since P_X is continuous and open, then G∈A = P_X{X~*spgcl*{y}}∉PO(X_0, $Q(X_0)$) and H∉A

Theorem 5.06: *The following are equivalent:*
(i) X is spgR_0 (ii) X_0 = {spgcl{x}: x∈X} and (iii) (X_0, $Q(X_0)$) is spgT_1

Proof: (i) ⇒ (ii) Let C∈X_0, and let x∈C. If y∈C, then y∈*spgcl*{y} = *spgcl*{x}, which implies C∈*spgcl*{x}. If y∈*spgcl*{x}, then x∈*spgcl*{y}, since, otherwise, x∈X~*spgcl*{y}∈PO(X,τ) which implies *spgcl*{x}⊂X~*spgcl*{y}, which is a contradiction. Thus, if y∈*spgcl*{x}, then x∈*spgcl*{y}, which implies *spgcl*{y} = *spgcl*{x} and y∈C. Hence X_0 = {*spgcl*{x}: x∈X}

(ii)⇒(iii) Let A ≠ B∈X_0. Then there exists x, y∈X such that A = *spgcl*{x}; B = *spgcl*{y}, and *spgcl*{x}∩*spgcl*{y} = ϕ. Then A∈C = P_X (X~*spgcl*{y})∈PO(X_0, $Q(X_0)$) and B∉C. Thus (X_0, $Q(X_0)$) is spg-T_1

(iii) ⇒ (i) Let x∈U∈ *αGO(X)*. Let y∉U and C_x, C_y ∈X_0 containing x and y respectively. Then x∉ *spgcl*{y}, which implies C_x ≠ C_y and there exists spg-open set A such that C_x∈A and C_y∉A. Since P_X is continuous and open, then y∈B = P_X^{-1}(A)∈ x∈*SPGO(X)* and x∉B, which implies y∉*spgcl*{x}. Thus *spgcl*{x}⊂*U*. This is true for all *spgcl*{x} implies ∩*spgcl*{x}⊂ *U*. Hence *X* is spg-R_0

Theorem 5.07: *(X, τ) is* spg-*R_1 iff (X_0, $Q(X_0)$) is* spg-*T_2*

The proof is straight forward from theorems 5.05 and 5.06 and is omitted

Theorem 5.08: *X is* spg-*T_i; i = 0,1,2. iff there exists a* spg-*continuous, almost–open, 1–1 function from (X, τ) into a* spg-*T_i space ; i = 0,1,2. respectively.*

Theorem 5.09: *If f: (X, τ)→ (Y, σ) is* spg-*continuous,* spg-*open, and x, y∈X such that spgcl{x} = spgcl{y}, then spgcl{f(x)} = spgcl{f(y)}.*

Theorem 5.10: *The following are equivalent*
(i) (X, τ) is spg-*T_0*
(ii) Elements of X_0 are singleton sets and
(iii)There exists a spg-*continuous,* spg-*open, 1–1 function f: (X, τ)→ (Y, σ), where (Y, σ) is* spg-*T_0*

Proof: (i) is equivalent to (ii) and (i) ⇒ (iii) are straight forward and is omitted.

(iii) ⇒ (i) Let x, y∈X such that *f*(x) ≠ *f*(y), which implies *spgcl*{*f*(x)} ≠ *spgcl*{*f*(y)}. Then by theorem 5.09, *spgcl*{x} ≠ *spgcl*{y}. Hence *(X, τ)* is spg-T_0

Corollary 5.11: *A space (X, τ) is* spg-*T_i ; i = 1,2 iff (X, τ) is* spg-*T_{i-1} ; i = 1,2, respectively, and there exists a* spg-*continuous ,* spg-*open, 1–1 function f: (X, τ) into a* spg-*T_0 space.*

Definition 5.04:*f*:*X*→Y is point–spg-closure 1–1 iff for x, y∈X such that *spgcl*{x} ≠ *spgcl*{y}, *spgcl*{*f*(x)} ≠ *spgcl*{*f*(y)}.

Theorem 5.12:
(i)If f: (X, τ)→ (Y, σ) is point– spg-*closure 1–1 and (X, τ) is* spg-*T_0, then f is 1–1*
(ii)If f: (X, τ)→ (Y, σ), where (X, τ)and (Y, σ) are spg-*T_0 then f is point–* spg-*closure 1–1 iff f is 1–1*

The following result can be obtained by combining results for spg-T_0– identification spaces, spg-induced functions and spg-T_i spaces; i = 1,2.

Theorem 5.13: *X is* spg-*R_i ; i = 0,1 iff there exists a* spg-*continuous , almost–open point–* spg-*closure 1–1 function f: (X, τ) into a* spg-*R_i space; i = 0,1 respectively.*

spg-Normal; Almost spg-normal and Mildly spg-normal spaces

Definition 6.1: A space X is said to be spg-normal if for any pair of disjoint closed sets F_1 and F_2 , there exist disjoint spg-open sets U and V such that F_1 ⊂ U and F_2 ⊂ V.

Example 4: Let X = {a, b, c} and τ = {ϕ, {a}, {b, c}, X}. Then X is spg-normal.

Example 5: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is *spg*-normal, normal and almost normal.

We have the following characterization of spg-normality.

Theorem 6.1: For a space X the following are equivalent:
(i) *X* is spg-normal.
(ii) For every pair of open sets *U* and *V* whose union is *X*, there exist spg-closed sets *A* and *B* such that *A*⊂*U*, *B* ⊂*V* and *A*∪*B* = *X*.
(iii) For every closed set *F* and every open set *G* containing *F*, there exists a spg-open set *U* such that *F*⊂*U*⊂*spgcl(U)*⊂*G*.

Proof: (i)⇒(ii): Let *U* and *V* be a pair of open sets in a spg-normal space *X* such that *X* = *U*∪*V*. Then *X*–*U*, *X*–*V* are disjoint closed sets. Since X is spg-normal there exist

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disjoint spg-open sets U_1 and V_1 such that $X–U\subset U_1$ and $X\text{-}V\subset V_1$. Let $A = X–U_1$, $B = X–V_1$. Then A and B are spg-closed sets such that $A\subset U$, $B\subset V$ and $A\cup B = X$.

(b) $\Rightarrow$(c): Let F be a closed set and G be an open set containing F. Then $X–F$ and G are open sets whose union is X. Then by (b), there exist spg-closed sets W_1 and W_2 such that $W_1 \subset X–F$ and $W_2 \subset G$ and $W_1\cup W_2 = X$. Then $F\subset X–W_1$, $X–G \subset X–W_2$ and $(X–W_1)\cap(X–W_2) = \phi$. Let $U = X–W_1$ and $V= X–W_2$. Then U and V are disjoint spg-open sets such that $F\subset U\subset X–V\subset G$. As $X–V$ is spg-closed set, we have $spgcl(U) \subset X–V$ and $F\subset U\subset spgcl(U)\subset G$.

(c) $\Rightarrow$ (a): Let F_1 and F_2 be any two disjoint closed sets of X. Put $G = X–F_2$, then $F_1\cap G = \phi$. $F_1\subset G$ where G is an open set. Then by (c), there exists a spg-open set U of X such that $F_1 \subset U \subset spgcl(U) \subset G$. It follows that $F_2 \subset X–spgcl(U) = V$, say, then V is spg-open and $U\cap V = \phi$. Hence F_1 and F_2 are separated by spg-open sets U and V. Therefore X is spg-normal.

Theorem 6.2: A regular open subspace of a spg-normal space is spg-normal.

Example 6: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is spg-normal and spg-regular.

However we observe that every spg-normal spg-R_0 space is spg-regular.

Definition 6.2: A function f:X $\to$ Y is said to be almost –spg-irresolute if for each x in X and each spg-neighborhood V of f(x), $spgcl(f^{-1}(V))$ is a spg-neighborhood of x.

Clearly every spg-irresolute map is almost spg-irresolute.

The Proof of the following lemma is straightforward and hence omitted.

Lemma 6.1: f is almost spg-irresolute iff $f^{-1}(V) \subset$ spg-int($spgcl(f^{-1}(V))$)) for every V$\in$ $SPGO$(Y).

Lemma 6.2: f is almost spg-irresolute iff $f(spgcl(U)) \subset spgcl(f(U))$ for every U$\in$ $SPGO$(X).

Proof: Let U$\in$ $SPGO$(X).Suppose y$\notin$ $spgcl(f(U))$. Then there exists V$\in$ $SPGO$(y) such that V$\cap f$(U) = ϕ. Hence $f^{-1}(V)\cap U = \phi$. Since U$\in$ $SPGO$(X), we have spg-int($spgcl(f^{-1}(V))$) $\cap$ $spgcl$(U) = ϕ. Then by lemma 6.1, $f^{-1}(V)\cap spgcl(U) = \phi$ and hence V$\cap f(spgcl(U)) = \phi$. This implies that y$\notin f(spgcl(U))$.

Conversely, if V$\in$ $SPGO$(Y), then W = X- $spgcl(f^{-1}(V))$$\in$ $SPGO$(X). By hypothesis, $f(spgcl(W))\subset spgcl(f(W))$ and hence X- spg-int($spgcl(f^{-1}(V))$) = $spgcl(W)\subset f^{-1}(spgcl(f(W)))\subset f(spgcl[f(X\text{-}f^{-1}(V))])\subset f^{-1}[spgcl(Y\text{-}V)] = f^{-1}(Y\text{-}V) = X\text{-}f^{-1}(V)$. Therefore, $f^{-1}(V)\subset$ spg-int($spgcl(f^{-1}(V))$). By lemma 6.1, f is almost spg-irresolute.

Now we prove the following result on the invariance of spg-normality.

Theorem 6.3: If f is an M-spg-open continuous almost spg-irresolute function from a spg-normal space X onto a space Y, then Y is spg-normal.

Proof: Let A be a closed subset of Y and B be an open set containing A. Then by continuity of f, f^{-1}(A) is closed and f^{-1}(B) is an open set of X such that $f^{-1}(A) \subset f^{-1}(B)$. As X is spg-normal, there exists a spg-open set U in X such that $f^{-1}(A) \subset U \subset spgcl(U)\subset f^{-1}(B)$. Then $f(f^{-1}(A))\subset f(U) \subset f(spgcl(U)) \subset f(f^{-1}(B))$. Since f is M-spg-open almost spg-irresolute surjection, we obtain A$\subset$ f(U) $\subset$ $spgcl(f(U)) \subset$ B. Then again by Theorem 6.1 the space Y is spg-normal.

Lemma 6.3: A mapping f is M-spg-closed if and only if for each subset B in Y and for each spg-open set U in X containing f^{-1}(B), there exists a spg-open set V containing B such that $f^{-1}(V)\subset U$.

Theorem 6.4: If f is an M-spg-closed continuous function from a spg-normal space onto a space Y, then Y is spg-normal.

Proof of the theorem is routine and hence omitted.

Now in view of lemma 2.2 [9] and lemma 6.3, we prove that the following result.

Theorem 6.5: If f is an M-spg-closed map from a weakly Hausdorff spg-normal space X onto a space Y such that f^{-1}(y) is S-closed relative to X for each y$\in$Y , then Y is spg-T_2.

Proof: Let y_1 and y_2 be any two distinct points of Y. Since X is weakly Hausdorff, $f^{-1}(y_1)$ and $f^{-1}(y_2)$ are disjoint closed subsets of X by lemma 2.2 [9]. As X is spg-normal, there exist disjoint spg-open sets V_1 and V_2 such that $f^{-1}(y_i) \subset V_i$, for i = 1,2. Since f is M-spg-closed, there exist spg-open sets U_1 and U_2 containing y_1 and y_2 such that $f^{-1}(U_i) \subset V_i$ for i = 1,2. Then it follows that $U_1\cap U_2 = \phi$. Hence Y is spg-T_2.

Theorem 6.6: For a space X we have the following:

(a) If X is normal then for any disjoint closed sets A and B, there exist disjoint spg-open sets U, V such that A $\subset$ U and B $\subset$ V;

(b) If X is normal then for any closed set A and any open set V containing A, there exists an spg-open set U of X such that A$\subset$U$\subset$$spgcl$(U) $\subset$V.

Definition 6.2: X is said to be almost spg-normal if for each closed set A and each regular closed set B such that A$\cap$B = ϕ, there exist disjoint spg-open sets U and V such that A$\subset$U and B$\subset$V.

Clearly, every spg-normal space is almost spg-normal, but not conversely in general.

Now, we have characterization of almost spg-normality in the following.

Theorem 6.7: For a space X the following statements are equivalent:

(i) X is almost spg-normal
(ii) For every pair of sets U and V , one of which is open and the other is regular open whose union is X, there exist spg-closed sets G and H such that G⊂U ,H⊂V and G∪H = X.
(iii) For every closed set A and every regular open set B containing A, there is a spg-open set V such that A ⊂V⊂ *spgcl*(V) ⊂ B.
Proof: (a)⇒(b) Let U be an open set and V be a regular open set in an almost spg-normal space X such that U∪V = X. Then (X-U) is closed set and (X-V) is regular closed set with (X-U)∩(X-V) = ϕ. By almost spg-normality of X, there exist disjoint spg-open sets U_1 and V_1 such that X-U ⊂ U_1 and X-V ⊂ V_1. Let G = X- U_1 and H = X-V_1. Then G and H are spg-closed sets such that G⊂U, H⊂V and G∪H = X.
(b) ⇒ (c) and (c) ⇒ (a) are obvious.
One can prove that almost spg-normality is also regular open hereditary.
Almost spg-normality does not imply almost spg-regularity in general. However, we observe that every almost spg-normal spg-R_0 space is almost spg-regular.
Theorem 6.8: Every almost regular, *spg*-compact space X is almost spg-normal.
Recall that a function *f*: X→ Y is called rc-continuous if inverse image of regular closed set is regular closed.
Now, we state the invariance of almost spg-normality in the following.
Theorem 6.9: If *f* is continuous M-spg-open rc-continuous and almost spg-irresolute surjection from an almost spg-normal space X onto a space Y, then Y is almost spg-normal.

Definition 6.3: A space X is said to be mildly spg-normal if for every pair of disjoint regular closed sets F_1 and F_2 of X, there exist disjoint spg-open sets U and V such that F_1 ⊂ U and F_2 ⊂ V.
Example 7: Let X = {a, b, c, d} with τ = {ϕ, {a}, {b}, {d}, {a, b}, {a, d}, {b, d}, {a, b, c}, {a, b, d}, X} is Mildly *spg*-normal.
We have the following characterization of mild spg-normality.
Theorem 6.10: For a space X the following are equivalent.
(i) X is mildly spg-normal.
(ii) For every pair of regular open sets U and V whose union is X, there exist spg-closed sets G and H such that G ⊂ U, H ⊂ V and G∪H = X.
(iii) For any regular closed set A and every regular open set B containing A, there exists a spg-open set U such that A⊂U⊂*spgcl*(U)⊂B.
(iv) For every pair of disjoint regular closed sets, there exist spg-open sets U and V such that A⊂U, B⊂V and *spgcl*(U)∩ *spgcl*(V) = ϕ.

This theorem may be proved by using the arguments similar to those of Theorem 6.7.
Also, we observe that mild spg-normality is regular open hereditary.
Definition 6.4: A space X is weakly spg-regular if for each point x and a regular open set U containing {x}, there is a spg-open set V such that x∈ V ⊂ clV ⊂ U.
Example 8: Let X = {a, b, c} and τ = {ϕ, {b},{a, b},{b, c}, X}. Then X is weakly *spg*-regular.
Example 9: Let X = {a, b, c} and τ = {ϕ, {a},{b},{a, b}, X}. Then X is not weakly *spg*-regular.
Theorem 6.11: If *f* : X → Y is an M-spg-open rc-continuous and almost spg-irresolute function from a mildly spg-normal space X onto a space Y, then Y is mildly spg-normal.
Proof: Let A be a regular closed set and B be a regular open set containing A. Then by rc-continuity of *f*, f^{-1}(A) is a regular closed set contained in the regular open set f^{-1}(B). Since X is mildly spg-normal, there exists a spg-open set V such that f^{-1}(A) ⊂V⊂ *spgcl*(V) ⊂ f^{-1}(B) by Theorem 6.10. As *f* is M-spg-open and almost spg-irresolute surjection, it follows that *f*(V)∈ *SPGO*(Y) and A⊂ *f*(V) ⊂ *spgcl*(*f*(V))⊂ B. Hence Y is mildly spg-normal.
Theorem 6.12: If *f*: X → Y is rc-continuous, M-spg-closed map from a mildly spg-normal space X onto a space Y, then Y is mildly spg-normal.

spg-US spaces

Definition 7.1:A sequence <x_n> is said to be *spg*-converges to a point x of X, written as <x_n> $\rightarrow^{spg}$ x if <x_n> is eventually in every *spg*-open set containing x.

Clearly, if a sequence <x_n> *r*-converges to a point x of X, then <x_n> *spg*-converges to x.

Definition 7.2:X is said to be *spg*-US if every sequence <x_n> in X *spg*-converges to a unique point.

Definition 7.3: A set F is sequentially *spg*-closed if every sequence in F *spg*-converges to a point in F.

Definition 7.4: A subset G of a space X is said to be sequentially *spg*-compact if every sequence in G has a subsequence which *spg*-converges to a point in G.

Definition 7.5: A point y is a *spg*-cluster point of sequence <x_n> iff <x_n> is frequently in every *spg*-open set containing x. The set of all *spg*-cluster points of $<x_n>$ will be denoted by *spg*-cl(x_n).

Definition 7.6: A point y is *spg*-side point of a sequence $<x_n>$ if y is a *spg*-cluster point of $<x_n>$ but no subsequence of $<x_n>$ *spg*-converges to y.

Definition 7.7: A space X is said to be
(i) *spg*-S_1 if it is *spg*-US and every sequence $<x_n>$ *spg*-converges with subsequence of $<x_n>$ *spg*-side points.
(ii) *spg*-S_2 if it is *spg*-US and every sequence $<x_n>$ in X *spg*-converges which has no *spg*-side point.

Using sequentially continuous functions, we define sequentially *spg*-continuous functions.

Definition 7.8: A function *f* is said to be sequentially *spg*-continuous at x $\in$ X if $f(x_n) \to^{spg} f(x)$ whenever $<x_n> \to^{spg}$ x. If *f* is sequentially *spg*-continuous at all x$\in$X, then *f* is said to be sequentially *spg*-continuous.

Theorem 7.1: We have the following:
(i) Every *spg*-T_2 space is *spg*-US.
(ii) Every *spg*-US space is *spg*-T_1.
(iii) X is *spg*-US iff the diagonal set is a sequentially *spg*-closed subset of X x X.
(iv) X is *spg*-T_2 iff it is both *spg*-R_1 and *spg*-US.
(v) Every regular open subset of a *spg*-US space is *spg*-US.
(vi) Product of arbitrary family of *spg*-US spaces is *spg*-US.
(vii) Every *spg*-S_2 space is *spg*-S_1 and Every *spg*-S_1 space is *spg*-US.

Theorem 7.2: In a *spg*-US space every sequentially *spg*-compact set is sequentially *spg*-closed.
Proof: Let X be *spg*-US space. Let Y be a sequentially *spg*-compact subset of X. Let $<x_n>$ be a sequence in Y. Suppose that $<x_n>$ *spg*-converges to a point in X-Y. Let $<x_{np}>$ be subsequence of $<x_n>$ that *spg*-converges to a point y $\in$ Y since Y is sequentially *spg*-compact. Also, let a subsequence $<x_{np}>$ of $<x_n>$ *spg*-converge to x $\in$ X-Y. Since $<x_{np}>$ is a sequence in the *spg*-US space X, x = y. Thus, Y is sequentially *spg*-closed set.

Theorem 7.3: Let *f* and *g* be two sequentially *spg*-continuous functions. If Y is *spg*-US, then the set A = {x | *f*(x) = *g*(x)} is sequentially *spg*-closed.
Proof: Let Y be *spg*-US and suppose that there is a sequence $<x_n>$ in A *spg*-converging to x $\in$ X. Since *f* and *g* are sequentially *spg*-continuous functions, $f(x_n) \to^{spg} f(x)$ and $g(x_n) \to^{spg} g(x)$. Hence *f*(x) = *g*(x) and x$\in$A. Therefore, A is sequentially *spg*-closed.

Sequentially sub-spg-continuity

In this section we introduce and study the concepts of sequentially sub-spg-continuity, sequentially nearly spg-continuity and sequentially spg-compact preserving functions and study their relations and the property of spg-US spaces.

Definition 8.1: A function *f* is said to be
(i) sequentially nearly spg-continuous if for each point x$\in$X and each sequence $<x_n> \to^{spg}$ x in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ such that $<f(x_{nk})> \to^{spg} f(x)$.
(ii) sequentially sub-spg-continuous if for each point x$\in$X and each sequence $<x_n> \to^{spg}$ x in X, there exists a subsequence $<x_{nk}>$ of $<x_n>$ and a point y$\in$Y such that $<f(x_{nk})> \to^{spg}$ y.
(iii) sequentially spg-compact preserving if *f*(K) is sequentially spg-compact in Y for every sequentially spg-compact set K of X.

Lemma 8.1: Every function *f* is sequentially sub-spg-continuous if Y is a sequentially spg-compact.
Proof: Let $<x_n> \to^{spg}$ x in X. Since Y is sequentially spg-compact, there exists a subsequence $\{f(x_{nk})\}$ of $\{f(x_n)\}$ spg-converging to a point y$\in$Y. Hence *f* is sequentially sub-spg-continuous.

Theorem 8.1: Every sequentially nearly spg-continuous function is sequentially spg-compact preserving.
Proof: Assume *f* is sequentially nearly spg-continuous and K any sequentially spg-compact subset of X. Let $<y_n>$ be any sequence in *f* (K). Then for each positive integer n, there exists a point $x_n \in$ K such that $f(x_n) = y_n$. Since $<x_n>$ is a sequence in the sequentially spg-compact set K, there exists a subsequence $<x_{nk}>$ of $<x_n>$ spg-converging to a point x $\in$ K. By hypothesis, *f* is sequentially nearly spg-continuous and hence there exists a subsequence $<x_j>$ of $<x_{nk}>$ such that $f(x_j) \to^{spg} f(x)$. Thus, there exists a subsequence $<y_j>$ of $<y_n>$ spg-converging to *f*(x)$\in$*f*(K). This shows that *f*(K) is sequentially spg-compact set in Y.

Theorem 8.2: Every sequentially pre-continuous function is sequentially spg-continuous.
Proof: Let *f* be a sequentially pre-continuous and $<x_n> \to^{p}$ x$\in$X. Then $<x_n> \to^{p}$ x. Since *f* is sequentially pre-continuous, $f(x_n) \to^{p} f(x)$. But we know that $<x_n> \to^{p}$ x implies $<x_n> \to^{spg}$ x and hence $f(x_n) \to^{spg} f(x)$ implies *f* is sequentially spg-continuous.

Theorem 8.3: Every sequentially spg-compact preserving function is sequentially sub-spg-continuous.

Proof: Suppose *f* is a sequentially spg-compact preserving function. Let x be any point of X and <x_n> any sequence in X spg-converging to x. We shall denote the set {x_n | n= 1,2,3, …} by A and K = A ∪ {x}. Then K is sequentially spg-compact since (x_n) $\rightarrow^{spg}$ x. By hypothesis, *f* is sequentially spg-compact preserving and hence *f*(K) is a sequentially spg-compact set of Y. Since {*f*(x_n)} is a sequence in *f*(K), there exists a subsequence {*f*(x_{nk})} of {*f*(x_n)} spg-converging to a point y∈*f*(K). This implies that *f* is sequentially sub-spg-continuous.

Theorem 8.4: A function *f*: X→ Y is sequentially spg-compact preserving iff $f_{/K}$: K → *f*(K) is sequentially sub-spg-continuous for each sequentially spg-compact subset K of X.

Proof: Suppose *f* is a sequentially spg-compact preserving function. Then *f*(K) is sequentially spg-compact set in Y for each sequentially spg-compact set K of X. Therefore, by Lemma 8.1 above, $f_{/K}$: K→ *f*(K) is sequentially spg-continuous function.

Conversely, let K be any sequentially spg-compact set of X. Let <y_n> be any sequence in *f*(K). Then for each positive integer n, there exists a point x_n∈K such that *f*(x_n) = y_n. Since <x_n> is a sequence in the sequentially spg-compact set K, there exists a subsequence <x_{nk}> of <x_n> spg-converging to a point x ∈ K. By hypothesis, $f_{/K}$: K→ *f*(K) is sequentially sub-spg-continuous and hence there exists a subsequence <y_{nk}> of <y_n> spg-converging to a point y∈ *f*(K).This implies that *f*(K) is sequentially spg-compact set in Y. Thus, *f* is sequentially spg-compact preserving function.

The following corollary gives a sufficient condition for a sequentially sub-spg-continuous function to be sequentially spg-compact preserving.

Corollary 8.1: If *f* is sequentially sub-spg-continuous and *f*(K) is sequentially spg-closed set in Y for each sequentially spg-compact set K of X, then *f* is sequentially spg-compact preserving function.

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